ASYMPTOTIC STRUCTURE OF SIMPLE COSMOLOGICAL MODELS

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The future null infinity for all non-contracting Robertson-Walker spacetimes is studied systematically. A theorem is proved which establishes the expected relation between the nature of $\mathscr{I}^+$ and the appearance or absence of cosmic event horizons.

1. INTRODUCTION

In the standard model of cosmology it is assumed that, on the largest scale, the Universe can be reasonably represented by a Robertson-Walker (R-W) spacetime.

When we observe the Universe we usually obtain information encoded in the electromagnetic radiation that arrives from the particular observed object. As we try to study more distant objects we are forced to direct our attention to earlier times. Thus when we are studying the largest scale of our Univers we are, for all practical purpose, at future null infinity of the observed region.

It is then necessary in the study of the largest scale of the Universe to have a clear picture of future null infinity of the Robertson-Walker spacetimes.

We have made a systematic study of future null infinity for non-contracting R-W spacetimes.

Some particular examples have been extensively discussed in the literature [1,2,3] already. We have however developed the techniques that allow the study of all non-contracting models.

In the remainder of this introductory section we will mainly present the notation to be used.

Let us consider non-contracting Robertson-Walker models. Their line element can be given by

$$ds^2 = \left(2\,r_0 - A^2\right) du^2 + 2\,du\,dr - A^2\, f_k^2\, d\Sigma^2$$

where $d\Sigma^2$ is the line element of the unit sphere, which can be expressed by

$$d\Sigma^2 = \frac{4\,d\zeta\, d\bar{\zeta}}{\left(1+\zeta\bar{\zeta}\right)^2} = \frac{d\zeta\, d\bar{\zeta}}{P_0^2}$$

with

$$P_0 = \frac{\left(1+\zeta\bar{\zeta}\right)}{2}$$

and the function f_x is defined by

$$f_k(x) = \begin{cases} \sinh(x) & \text{for } K=-1 \quad 0 \le x < \infty \\ x & \text{for } K=0 \quad 0 \le x < \infty \\ \sin(x) & \text{for } K=1 \quad 0 \le x \le \pi \end{cases}$$

The range of the coordinate t is associated with the behavior of the function A(t). When there is an initial singularity followed by a continuing expansion one takes $0 < t < \infty$.

By a non-contracting R-W space we mean that the scalar A(t) must satisfy

$$\frac{\partial A}{\partial t} \ge 0$$

The relation between the coordinates (u,x) and the standard time coordinate t of Robertson-Walker spacetimes can be given by

$$du = \frac{dt}{A(t)} - dx$$

2. THE THEOREM

We are therefore confronted with the question of how to choose an appropirate conformal factor that will make the conformal metric regular (meaning al least continuous) at future null infinity. Of course this is associated with the behavior of the function A(t) which is the only nontrivial input in the metric.

Let us be more precise in our discussion. We will the manifold $\mathscr{I}^+$ to be future null infinity of a non-contracting C^∞ Robertson-Walker spacetime (M, g_{ab}) if there exists a manifold $\tilde{M}$ with boundary $\mathscr{I}^+$, metric $\tilde{g}_{ab}$ and a function Ω on $\tilde{M}$ such that a neighborhood of $\tilde{\mathscr{I}}^+$ in $\tilde{M}$ is diffeomorphic to a neighborhood of $\mathscr{I}^+$ in the manifold $M \cup \mathscr{I}^+$ and

a) on M: Ω is C^∞, $\Omega > 0$ and $\tilde{g}_{ab} = \Omega^2 g_{ab}$;

b) al $\mathscr{I}^+$: $\Omega = 0$, Ω is C^0; at every point of $\mathscr{I}^+$ there end future derected null geodesics of $\tilde{M}$, and $\tilde{g}_{ab}$ is non-degenerate.

Note that we are implicity requiring the conformal metric to be C^1 at scri. Also it should be observed that nothing is said about the differentiability properties of Ω at scri, that is we only require it to be continuous.

At this point it is important to recall that if v is an affine parameter along the null geodesics contained in the null hypersurfaces u = constant, but with respect to the conformal metric g_{ab}, then it can be related to r by the equation

$$\frac{\partial v}{\partial r} = -\Omega^2$$

Defining x_∞ by

$$x_\infty = \int_{t_0}^{\infty} \frac{dt'}{A(t')}$$

we want to know whether x_∞ is bounded (finite) or unbounded (infinite). We can then classify the R-W spacetimes in two classes such that:

Class B corresponds to x_∞ bounded and

Class U corresponds to x_∞ unbounded.

Class B coincides with the appearance for each observer of an event horizon; since for example an observer traveling along the geodesic x = const. > x_∞, θ = const., and φ = const. will never be able to get information from the events with $t >_0 t$ and $x < x_\infty$. The case U refers to those models where there is no event horizon.

It is observed that although we are forced to take the conformal factor as the inverse of the luminosity distance, in order to obtain a regular metric at scri, this conformal factor is not suitable for the study of the naure of scri, since we have seen that $d\Omega$ is sometimes zero or not defined at scri.

However we can choose the coordinate v such that v=0 at g^+ by appropiately defining $v_0(u)$; and by doing so we can use dv for the study of nature of g^+, since obviously v is a regular function at scri. In fact one finds that

$$\tilde{g}(dv, dv) = \frac{1}{f_k^2}$$

so scri is non-timelike, and furthermore in case U it is null and in case B spacelike.

We have just proved the following theorem,

THEOREM:

The future null infinity of non-contracting Robertson-Walker models is null or spacelike according to use absence or presence of cosmic event horizons respectively

This completes the statements appearing in the literature [3] which claimed, based on intuitive arguments, that when g^+ is null one expects no event horizons, and when g^+ is spacelike each observer will be assigned an event horizon. It is important to emphasize that although the same intuitive arguments were used for the nature of past null infinity, which normally coincides with the initial cosmic singularity, the analog theorem is not true. For examle the K = -1 de Sitter model would violate it, since past null infinity of de Sitter space (wich is spacelike) does not coincide with the inicial null cone cosmic singularity of the K = -1 model (wich does not possess particle horizons).

REFERENCES

1. S. W. Hawking and G. F. R. Ellis, The Large Scale Structure of Spacetime; Cambridge University Press, (1973).
2. R. Penrose in: Battelle Rencontres; ed. by C. DeWitt and J. A. Wheeler, W. A. Benjamin Inc., (1968).
3. R. Penrose in: Relativity, Groups and Topology; ed. by C. DeWitt and B. DeWitt, Gordon and Breach Science Publishers, (1964).