THE KELVIN-HELMHOLTZ INSTABILITY AT THE TERRESTRIAL MAGNETOPAUSE

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Se aplica nuestra teoría de la inestabilidad de Kelvin-Helmholtz compresible en presencia de cizalla magnética y diferencias de densidad y temperatura de ambos lados de la discontinuidad de velocidad, para estudiar la estabilidad de configuraciones típicas de la magnetopausa terrestre. Se muestra que la inclusión de los efectos de la compresibilidad lleva a desestabilizar el plasma para bajas velocidades relativas en situaciones que son estables en el límite incompresible, lo que muestra que la teoría incompresible da predicciones incorrectas acerca de la estabilidad

Our theory of the compressible Kelvin-Helmholtz instability for configurations with magnetic shear and density and temperature jumps across a velocity discontinuity is applied to study the stability in typical cases at the terrestrial magnetopause. It is shown that the inclusion of compressibility effects tends to destabilize plasmas for low relative velocities in problems that are stable in the incompressible limit. This shows that the incompressible theory is not reliable in these cases.

I. INTRODUCTION

The Kelvin-Helmholtz Instability (KHI) occurs at the terrestrial magnetopause, the boundary that separates the interplanetary magnetic field (IMF) present in the magnetosheath, and the geomagnetic field inside the magnetosphere. The relative motion is generated by the solar wind flowing through the magnetosheath. The flow velocity and the magnetic field are assumed parallel to the transition layer, and the magnetic field may change its direction (magnetic shear) and its magnitude across the transition. The low frequency magnetohydrodynamic (MHD) modes shall be our main concern in this paper since their instability may produce large-scale turbulence. The KHI is considered a major source of anomalous transport of momentum from the solar wind into the magnetosphere, and a cause of viscous drag at the magnetopause. This is so particularly during periods of northward IMF when the reconnection of magnetic lines is less likely to occur. We shall not consider here other unstable modes that can also arise close to, and inside the magnetopause, within frequency ranges higher than those treated by MHD and lead to microscopic turbulence.

The theory of the KHI in the case of arbitrary geometry is exceedingly complicated, but fortunately in many circumstances, the wavelength of the perturbation is very small as compared to the curvature radii of the transition layer. In these cases, the problem can be treated in a plane slab geometry, in which the unperturbed quantities (density ρ, pressure p, magnetic field $\boldsymbol{B}$ and mass flow velocity $\boldsymbol{u}$) depend only on the y coordinate (perpendicular to the transition layer). If the wavelength of the perturbation is large as compared to the thickness of the transition layer, we can ignore the structure inside the transition.

A substantial contribution to the understanding of the compressibility effects on the Kelvin-Helmholtz instability was given by Miura and Pritchett[1]. However, these effects have not yet been fully explored for general configurations with density and temperature discontinuities and magnetic field shear, such as those that occur in the magnetopause. In another paper in this volume[2] we analyze the stability of a discontinuity surface separating two uniform flows, which can be used for configurations of any kind. We show there that the problem involves *seven* independent parameters. Five of them characterize the plasma configuration; they can be taken as the ratios $r_b = B_2 / B_1$, $r_s = S_2 / S_1$ and $r_d = \rho_2 / \rho_1$, the relative velocity u, and the angles φ_1 from $\boldsymbol{B}_1$ to $\boldsymbol{u}$ and θ from $\boldsymbol{B}_1$ to $\boldsymbol{B}_2$ (magnetic shear angle). The remaining is the angle ψ_1 from $\boldsymbol{B}_1$ to $\boldsymbol{k}_t$, that identifies the perturbation we are considering. In addition, one has the condition

$$u_k \equiv u \cos\alpha = u \cos(\psi_1 - \varphi_1) \quad (1)$$

We have shown that the problem may be analyzed using diagrams in which we plot the growth rate $\operatorname{Im}(\nu)$ of the unstable modes as functions of ψ_1 and u_k, keeping fixed the remaining parameters.

II. APLICATIONS TO THE MAGNETOPAUSE: PHYSICAL PARAMETERS

By convention we shall always assume that the outside region (as seen from the Earth) is our region 1. In the applications, the geometrical and kinematical pa-

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rameters of the configuration, i. e. the flow velocities $\mathbf{u}_1'$ and $\mathbf{u}_2'$, and the angles $\psi_{B_1,u_1'}$ and $\psi_{B_1,u_2'}$ from $\mathbf{B}_1$ to $\mathbf{u}_1'$ and $\mathbf{u}_2'$, respectively, are given in a reference frame at rest with respect of the Earth. In our theory we use a reference frame moving with the average velocity $\mathbf{u}' = (\mathbf{u}_1' + \mathbf{u}_2')/2$, then we have $\mathbf{u}_1 = \mathbf{u}$, $\mathbf{u}_2 = -\mathbf{u}$, with $\mathbf{u} = (\mathbf{u}_1' - \mathbf{u}_2')/2$. In the following, all velocities are given in km/s. We find

$$u = \tfrac{1}{2}\sqrt{u_1'^2 + u_1'^2 - 2u_1'u_2'\cos\theta_u}\ , \qquad (2)$$

$$u' = \tfrac{1}{2}\sqrt{u_1'^2 + u_1'^2 + 2u_1'u_2'\cos\theta_u}\ , \qquad (3)$$

$$\theta_u = \psi_{B_1,u_2'} - \psi_{B_1,u_1'} \qquad (4)$$

$$\varphi_1 = \psi_{B_1-u_1'} + \mathrm{sign}(\sin\theta_u)\arccos\left(\frac{u_1' - u_2'\cos\theta_u}{2u}\right) \qquad (5)$$

The angle δ from $\boldsymbol{u}$ to $\boldsymbol{u}'$ is given by

$$\delta = \mathrm{sign}(\sin\theta_u)\arccos\left(\frac{u_1'^2 - u_2'^2}{4uu'}\right) \qquad (6)$$

Finally, the phase velocity of the perturbation in the Earth's frame is given by

$$v' = v - u'\cos(\psi_1 - \varphi_1 - \delta) \qquad (7)$$

At the flanks of the magnetopause, the magnetosheath flow becomes supersonic and the effects of compressibility on the KH instability are expected to be important. We shall examine a configuration at the near equatorial magnetopause at dusk, observed by Interball/tail during the event of January 11, 1997. This was at the tail of a coronal mass ejection passing Earth, during which a high density solar wind dynamic pressure strongly compressed the magnetosphere. As a consequence, Interball/tail was for some time in the magnetosheath, and reentered the magnetosphere, after the dynamic pressure returned to lower values. During the crossing of the magnetopause, from magnetosheath to low latitude boundary layer, the spacecraft measured the physical parameters necessary for a stability analysis. A previous paper[3] discusses the Kelvin-Helmholtz instability for this event, based on an incompressible model, and of other plasma waves at that locale. We intend to revisit that configuration with our compressible theory. We shall consider is the interface between the magnetosheath and the low latitude boundary layer.

In the first line of Table 1 we give the parameters of the event. The second line contains the parameters of a hypothetical configuration with a slightly larger value of $M_{A,1}$, to ascertain the effect of this change.

For the application of the theory it is necessary to take into account that owing to measurement uncertainties, the spacecraft data do not comply with the equilibrium condition at the interface. To emend this flaw, we replace the measured r_s (that is the less reliable datum) by the value derived using the remaining data in the equilibrium condition, given by

$$2M_{A,1}^2(1 - r_d r_s^2) = \gamma M_{S,1}^2(r_b^2 - 1)\,. \qquad (8)$$

The expressions of u, u', φ_1, δ and v' are given by eqs. (2)-(7), and the remaining quantities needed are the Alfven (A_i) and sound velocities (S_i) in each region $A_1 = u_1' / M_{A,1}$, $A_2 = A_1 r_b \sqrt{r_d}$, $S_1 = u_1' / M_{S,1}$, $S_2 = r_s S_1$

TABLE 3: PARAMETERS OF THE JANUARY 11, 1997 EVENT (SPACECRAFT DATA, VELOCITIES ARE EXPRESSED IN KM/S).

Case	u'_1	u'_2	$M_{A,1}$	$M_{S,1}$	r_b	r_d	ψ_{B_1,u_1}	ψ_{B_1,u_2}	θ
(a)	300	105	5	2	1.5	0.117	–100°	–120°	–80°
(b)	300	105	5.7	2	1.5	0.117	–100°	–120°	–80°

TABLE 4: CALCULATED PARAMETERS OF THE JANUARY 11, 1997 EVENT (VELOCITIES ARE EXPRESSED IN KM/S).

Case	u	φ_1	u'	δ	r_s	A_1
(a)	102.3	–89.9°	200.1	–15.3°	2.67	60
(b)	102.3	–89.9°	200.1	–15.3°	2.67	53

III. A CONFIGURATION IN THE NEAR FLANK OF THE MAGNETOPAUSE

The stability diagram corresponding to these parameters was shown in a previous paper[2], in which it can be appreciated that this configuration is stable according to the IMHD model, but is unstable when compressibility is taken into account. Moreover, this instability corresponds to the secondary modes, which have no counterpart in IMHD. Their growth rates are small, but significant, as can be appreciated in Fig. 1.

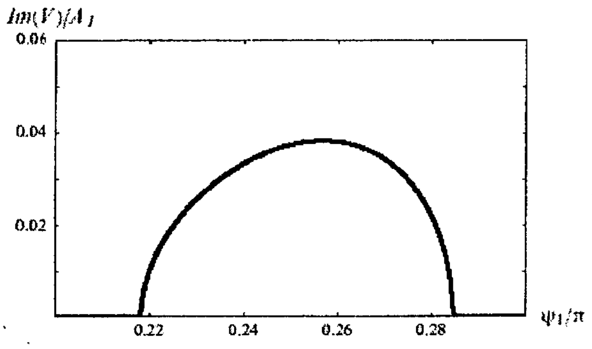

Fig. 1. Growth rate of the secondary modes for case (a).

When the line C of the configuration passes close to the marginal stability line for the primary modes, as happened in this case, the stability properties of the configuration are very sensitive to slight changes of the parameters. This can be appreciated in Fig. 2, in which we represent the growth rate of the instability, for a slightly different orientation of the relative velocity ***u*** with respect to the magnetic field ($\varphi_1 = 75.5°$ instead of 89.9°). This change produces a fourfold increase of the growth rate of the secondary instability (and a shift towards larger ψ_1). The primary instability is also present, although its growth rate is much smaller than that of the secondary modes. Note that this modified configuration is stable according to IMHD.

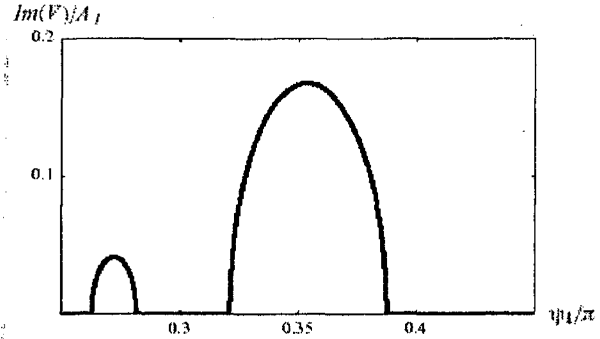

Fig.2. Effect of a change of the orientation of the relative velocity with respect to the magnetic field. The figure represents the growth rate for case (a), with $\varphi_1 = 75.5°$ instead of 89.9°. Note the primary modes centered around $\psi_1 \approx 0.27\pi$.

If we change the relative velocity, keeping the remaining parameters fixed, as if the solar wind had been slightly faster at that event, the line C of Fig. 1 of González and Gratton[2] shifts upwards, and the main instability becomes dominant for a 30% (or larger) increase of *u*.

The stability diagram for case (b), where $M_{A,1}$ differs from case (a) by 14% shows that the effect of this change is to increase the growth rate of the secondary instability with respect to the results of Fig.1. This modified configuration is stable according to IMHD.

IV. CONCLUSIONS

Compressibility should be taken into account when assessing the stability properties of configurations at the flanks of the magnetopause. In these cases the occurrence of the secondary instability and the shift of the boundary of the primary instability play an important role. Then, configurations that are stable if compressibility is neglected turn out to be unstable when it is considered. In these situations, the stability properties are quite sensitive on the values of the parameters. The estimates based on IMHD may be misleading, and a careful analysis is required in each case, since no simple rule of thumb can be given.

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